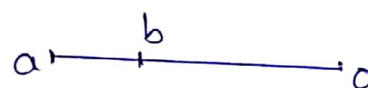
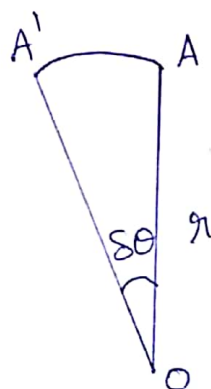
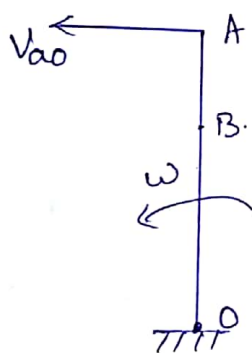


Velocity Analysis & Acceleration Analysis.

The velocity analysis is the prerequisite for acceleration analysis which further leads to force analysis of various links of a mechanism.

Motion of a link:-

Let a rigid link OA, of length  $r$ , rotate about a fixed point O with a uniform angular velocity  $\omega$  rad/sec in the counter-clockwise direction. OA turns through a small angle  $\delta\theta$  in a small interval of time  $\delta t$ . Then A will travel along the arc AA'.



velocity of A relative to O =  $\frac{\text{Arc AA'}}{\delta t}$

$$V_{ao} = \frac{r \delta\theta}{\delta t}$$

when  $\delta t \rightarrow 0$

$$V_{ao} = r \frac{d\theta}{dt}$$

$$V_{ao} = r\omega$$

As  $st$  approaches zero ( $st \rightarrow 0$ ),  $AA'$  will be perpendicular to  $OA$ . Thus, velocity of  $A$  is  $wa$  and is perpendicular to  $OA$ .

Consider a point  $B$  on the link  $OA$ .

velocity of  $B = \omega \cdot OB$  perpendicular to  $OB$ .

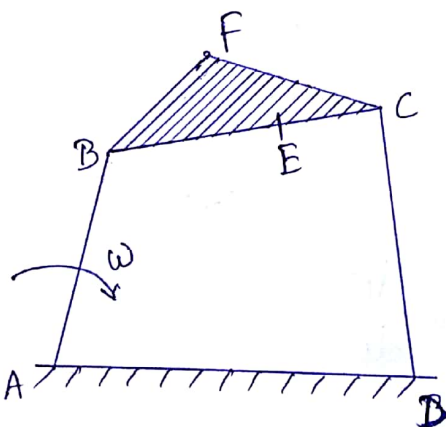
If  $ob$  represents the velocity of  $B$ ,

$$\frac{ob}{oa} = \frac{\omega OB}{\omega OA} = \frac{OB}{OA}$$

i.e.  $b$  divides the velocity vector  $oa$  in the same ratio as  $B$  divides the link.

### Four-Link Mechanism:-

Consider a four-link mechanism  $ABCD$  in which  $AD$  is fixed link and  $BC$  is the coupler.  $AB$  is the driver rotating at an angular speed of  $\omega$  rad/sec in the clockwise direction if it is a crank or moving at this angular velocity at this instant if it is a rocker.



The velocity of any point relative to any<sup>2</sup> other point on a fixed link is always zero. Thus, all the points on a fixed link are represented by one point in the velocity diagram.

Points A & D, both lie on fixed link AD.  
 $\therefore$  velocity of C relative to A is same as velocity of C relative to D.

velocity of C rel. to A = vel. of C rel. to B + vel. of B rel. to A

$$V_{ca} = V_{cb} + V_{ba}$$

$$V_{ca} = V_{cd} = V_{ba} + V_{cb}$$

$$dc = ab + bc$$

$V_{ba} = ab = \omega \cdot AB$  ; perpendicular to AB.

$V_{cb} = bc$  ;  $\perp^{\text{a}}$  to BC.

$V_{cd} = dc$  ;  $\perp^{\text{a}}$  to DC.

velocity diagram construction:-

- (1) Take the first vector ab as it is completely known.
- (2) To add vector bc to ab, draw a line  $\perp$  Bc through b, of any length. Since the direction-sense of bc is unknown, it can lie on



either side of  $b$ . A convenient length of the line can be taken on both sides of  $b$ .

- (3) Through  $d$ , draw a line  $\perp DC$  to locate the vector  $dc$ . The intersection of this line with the line of vector  $bc$  locates the point  $c$ .
- (4) Mark arrowheads on the vectors  $bc$  &  $dc$  to give the proper sense. Then  $dc$  is the magnitude and also represents the direction of the velocity of  $c$  relative to  $A$  ( $\vec{v}_{c/A}$  or  $\vec{v}_{c/D}$ ). It is also the absolute velocity of the point  $c$ .

(5) Intermediate point: The velocity of an intermediate point on any of the links can be found easily by dividing the corresponding velocity vector in the same ratio as the point divides the link.

For point  $E$  on the link  $BC$ ,

$$\frac{be}{bc} = \frac{BE}{BC}$$

$ae$  represents the absolute velocity of  $E$ .

(6) Offset point:

$$v_{fb} + v_{ba} = v_{fc} + v_{cd}$$

$$v_{ba} + v_{fb} = v_{cd} + v_{fc}$$

$$ab + bf = dc + cf$$



The vectors  $v_{ba}$  &  $v_{cd}$  are already there <sup>(3)</sup> on the velocity diagram.

$v_{fb}$  is  $\perp BF$ ; draw a line  $\perp BF$  through  $b$ .

$v_{fc}$  is  $\perp CF$ ; draw a line  $\perp CF$  through  $c$ .

The intersection of the two lines locates the point  $f$ .

$af$  &  $df$  indicates the velocity of  $F$  relative to  $A$  ( $af$ ) & the absolute velocity of  $F$ .

Angular velocities of links:-

Angular velocity of  $BC$ :-

(a) velocity of  $c$  relative to  $B$ ,  $v_{cb} = bc$ .

Point  $c$  relative to  $B$  moves in the direction-sense given by  $v_{cb}$  (upwards). Thus,  $c$  moves in the counter-clockwise direction about  $B$ .

$$v_{cb} = \omega_{cb} \times BC = \omega_{cb} \times CB.$$

$$\Rightarrow \omega_{cb} = \frac{v_{cb}}{CB}.$$

(b) velocity of  $B$  relative to  $c$ ,  $v_{bc} = cb$ .

$B$  relative to  $c$  moves in a direction-sense given by  $v_{bc}$  (downwards, opposite to  $bc$ ), i.e.,  $B$  moves in the counter-clockwise

direction about C with

$$\omega_{bc} = \frac{V_{bc}}{BC}$$

$\therefore \omega_{cb} = \omega_{bc}$  as  $V_{cb} = V_{bc}$  and the direction of rotation is the same.

$\therefore$  angular velocity of a link about one extremity is the same as the angular velocity about the other. In general, the angular velocity of link BC is  $\omega_{bc}$  ( $=\omega_{cb}$ ) in the counter-clockwise direction.

Angular velocity of CD:-

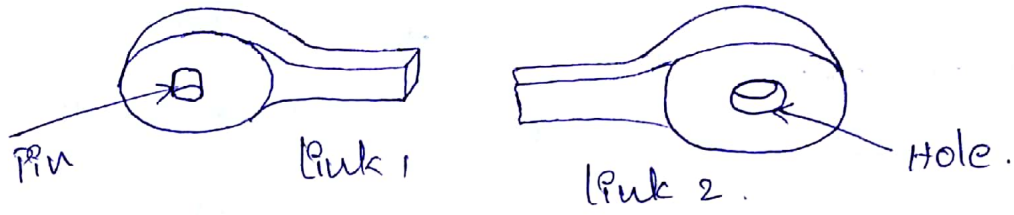
velocity of C relative to D,

$$V_{cd} = dc$$

$$\omega_{cd} = \frac{V_{cd}}{CD}$$

Velocity of Rubbing:-

Consider two ends of the two links of a turning pair. A pin is fixed to one of the links whereas a hole is provided in the other to fit the pin. When these links are joined, the surface of hole will rub on the surface of the pin. The rubbing velocity of the two surfaces will depend upon the angular velocity of a link relative to the other.



Pin at A:- The pin at A joints Links AD & AB. AD is fixed link, the velocity of rubbing will depend upon the angular velocity of AB only.

let  $r_a =$  radius of ~~pin~~ pin at A.

velocity of rubbing =  $r_a \cdot \omega_{ba}$

Pin at D:- let  $r_d =$  radius of pin at D.

velocity of rubbing =  $r_d \cdot \omega_{cd}$

Pin at B:

$\omega_{ba} = \omega_{ab} = \omega$  clockwise.

$\omega_{bc} = \omega_{cb} = \frac{v_{bc}}{BC}$  counter-clockwise.

let  $r_b =$  radius of pin at B.

velocity of rubbing =  $r_b (\omega_{ab} + \omega_{bc})$ .

Pin at C:-  $\omega_{bc} = \omega_{cb}$  counter-clockwise

$\omega_{dc} = \omega_{cd}$  clockwise

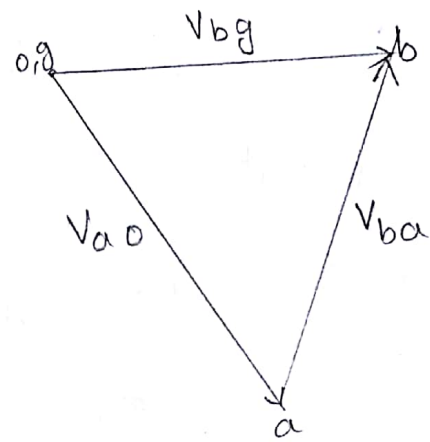
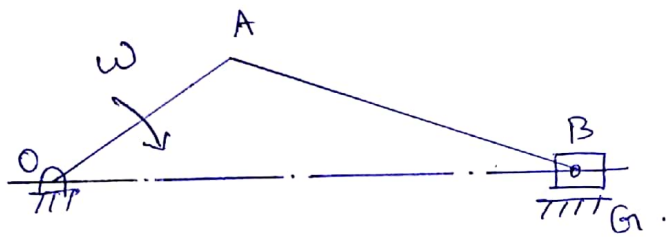
let  $r_c =$  radius of pin at C.

velocity of rubbing =  $r_c (\omega_{bc} + \omega_{dc})$ .



## Slider-Crank Mechanism:-

Consider a slider-crank mechanism in which  $OA$  is the crank with uniform angular velocity  $\omega$  rad/s in clockwise direction. At point  $B$ , a slider moves on the fixed guide  $G$ .  $AB$  is the coupler joining  $A$  &  $B$ .



velocity of  $B$  rel. to  $O$  = vel. of  $B$  rel. to  $A$  + vel. of  $A$  rel. to  $O$

$$v_{bo} = v_{ba} + v_{ao}$$

$$v_{bg} = v_{ao} + v_{ba}$$

$$g_b = o_a + a_b$$

Take the vector  $v_{ao}$  which is completely known.

$$v_{ao} = \omega \cdot OA \perp \text{to } OA.$$

$v_{ba}$  is  $\perp AB$ , draw a line  $\perp AB$  through  $a$ .

Through  $g$  or  $a$ , draw a line parallel to the motion of  $B$ .

The intersection of the two lines locates the point  $b$ .  $v_b$  indicates the velocity of slider B relative to guide G.

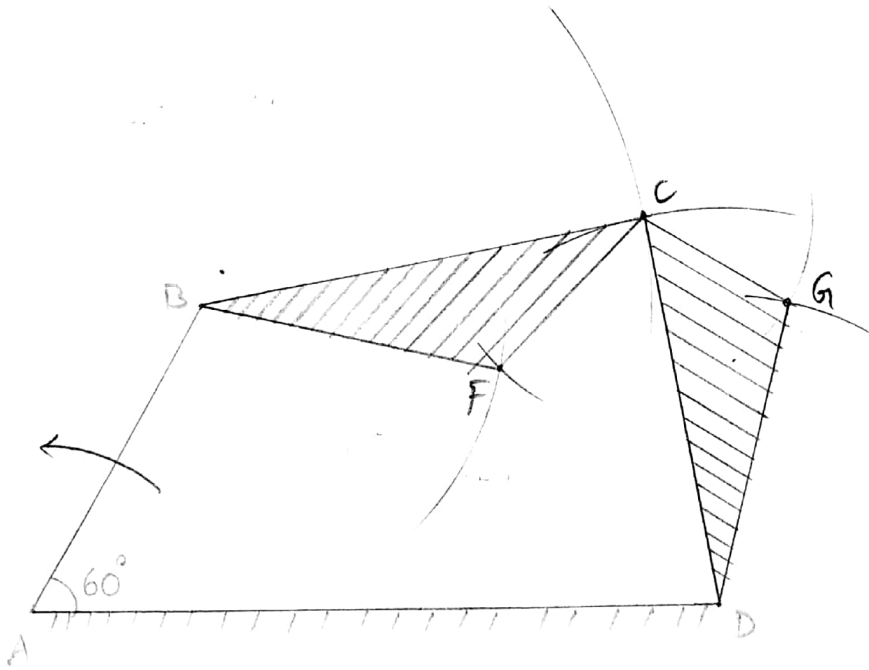
The coupler AB has angular velocity  $\omega$  in the counter-clockwise direction.

$$\omega_{ba} = \frac{v_{ba}}{AB}$$

- Pb:-** In a four-link mechanism, the dimensions of the links are as  $AB = 50$  mm,  $BC = 66$  mm,  $CD = 56$  mm and  $AD = 100$  mm. At the instant when  $\angle DAB = 60^\circ$ , the link AB has an angular velocity of  $10.5$  rad/s in the counter-clockwise direction. Determine
- velocity of point C
  - velocity of point E on the link BC when  $BE = 40$  mm
  - angular velocities of links BC & CD.
  - velocity of an offset point F on the link BC if  $BF = 45$  mm,  $CF = 30$  mm and BCF is read clockwise.
  - velocity of an offset point G on the link CD if  $CG = 24$  mm,  $DG = 44$  mm and DCG is read clockwise.
  - velocity of rubbing at pins A, B, C & D when the radii of the pins are  $30, 40, 25$  and  $35$  mm respectively.

Sol:-

A



(i) velocity of C,  $v_c = dc = 0.4 \text{ m/s}$ .

(ii)  $\frac{be}{bc} = \frac{BE}{BC}$

$be = 0.34 \times \frac{40}{66} = 0.206 \text{ m/s}$

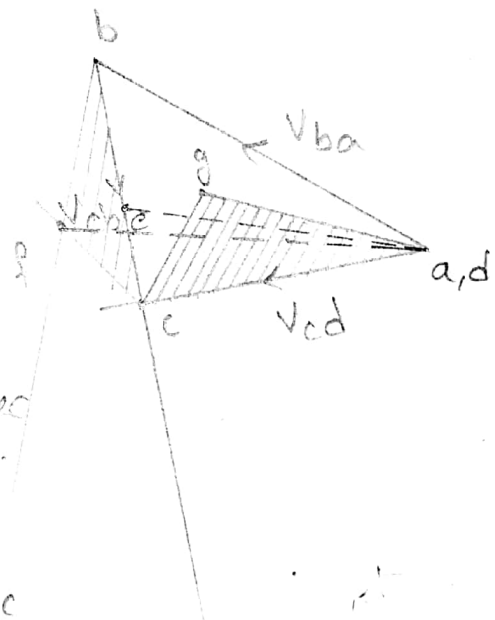
velocity e,  $ae = 0.41 \text{ m/s}$ .

(iii)  $\omega_{BC} = \frac{v_{cb}}{CB} = \frac{0.34}{66 \times 10^{-3}} = 5.15 \text{ rad/sec}$

(iv)  $\omega_{CD} = \frac{v_{cd}}{CD} = \frac{0.4}{56 \times 10^{-3}} = 7.14 \text{ rad/sec}$

(v) velocity of f,  $v_f = 0.52 \text{ m/sec}$

(vi) velocity of ~~sub~~ g,  $v_g = 0.33 \text{ m/sec}$





vel. of c rel. to A = vel. of c rel. to B + vel. of B rel. to A

$$V_{ca} = V_{cb} + V_{ba}$$

$$V_{cd} = V_{ba} + V_{cb}$$

$$dc = ab + bc$$

$$V_{ba} = AB \times \omega_{ba} = 0.05 \times 10.5 = 0.525 \text{ m/s.}$$

$V_{cb}$  is  $\perp BC$ , draw a line  $\perp BC$  through b.

$V_{cd}$  is  $\perp DC$ , draw a line  $\perp DC$  through d.

The intersection of the two lines locates the point c.

$$V_{cd} = V_{ba} + V_{cb}$$

$$dc = a$$

(i)  $V_c = ac = dc = 0.53 \text{ m/s}$

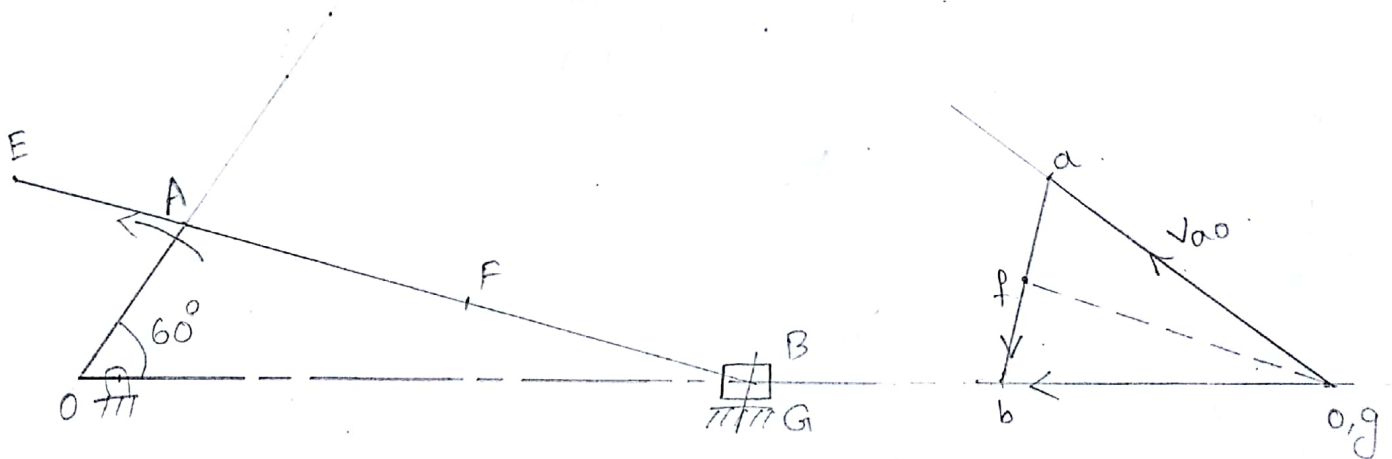
(ii) locate the point e on bc.

$$\frac{be}{bc} = \frac{BE}{BC}$$

Pb:- In a slider-crank mechanism, the crank is 480 mm long and rotates at 20 rad/s in the CCW direction. The length of the connecting rod is 1.6 m. When the crank turns ~~through~~  $60^\circ$  from the inner-dead centre, determine the

- (i) velocity of the slider
- (ii) velocity of the point E located at a distance 450 mm on the connecting rod extended.
- (iii) position and velocity of a point F on the connecting rod having the least absolute velocity
- (iv) angular velocity of the connecting rod.
- (v) velocities of rubbing at the pins of the crankshaft, crank and the cross-head diameters 80, 60 and 100 mm, respectively.

Sol:-



$$v_{ao} = \omega_{ao} \times OA = 20 \times 0.48 = 9.6 \text{ m/s.}$$

$$v_{bo} = v_{ba} + v_{ao}$$

$$v_{bg} = v_{ao} + v_{ba}$$

$$gb = oa + ab.$$

$v_{ba}$  is  $\perp$  AB, draw a line  $\perp$  AB through a.

The slider B has a linear motion relative to the guide G. Draw a line parallel to the direction of motion of the slider through g.

Intersection of these two lines will give the point b.

(i) velocity of slider,  $v_b = ob = gb = 9.7 \text{ m/s.}$

(ii) locate the point e on ba such that

$$\frac{ae}{ba} = \frac{AE}{BA}$$

$$ae = ba \times \frac{AE}{BA}$$

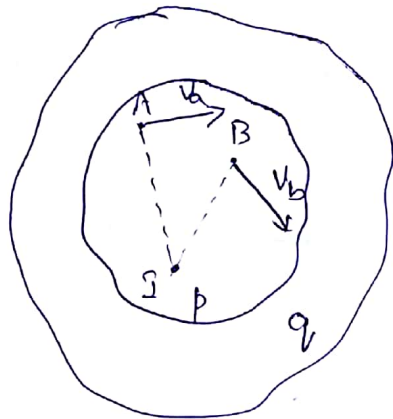
$\therefore v_e$

$$v_e =$$



## Instantaneous Centre (I-centre) :-

consider a plane body  $P$  having a non-linear motion relative to another plane body  $Q$ . At any instant, the linear velocities of two points  $A$  and  $B$  on the body  $P$  are  $v_A$  &  $v_B$  respectively.

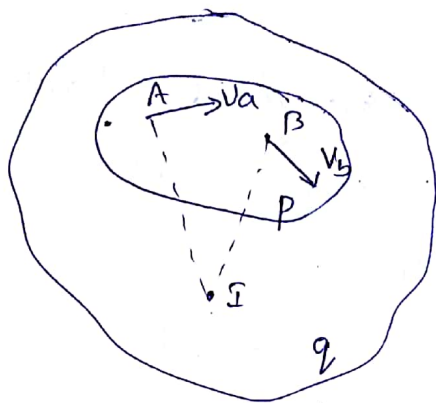


If a line is drawn perpendicular to the direction of  $v_A$  at  $A$ , the body can be imagined to rotate about some point on this line. Similarly, the centre of rotation of the body also lies on a line perpendicular to the direction of  $v_B$  at  $B$ . If the intersection of the two lines is at  $I$ , the body  $P$  will be rotating about  $I$  at that instant. This point  $I$  is known as the instantaneous centre of velocity or more commonly instantaneous centre of rotation for the body  $P$ .

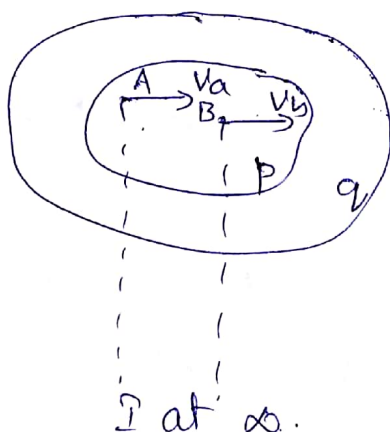
In case the perpendiculars to  $v_A$  and  $v_B$  at  $A$  &  $B$  respectively meet outside the body  $P$ , the  $I$ -centre will lie outside the body  $P$ .

Acc

Ac  
H  
d



If the directions of  $v_A$  and  $v_B$  are parallel and the perpendiculars at A and B meet at infinity, the I-centre of the body lies at infinity. This happens when the body is in linear motion.



Number of I-centres:- For two bodies having relative motion between them, there is an I-centre. In a mechanism, the number of I-centres will be equal to possible pairs of bodies or links.

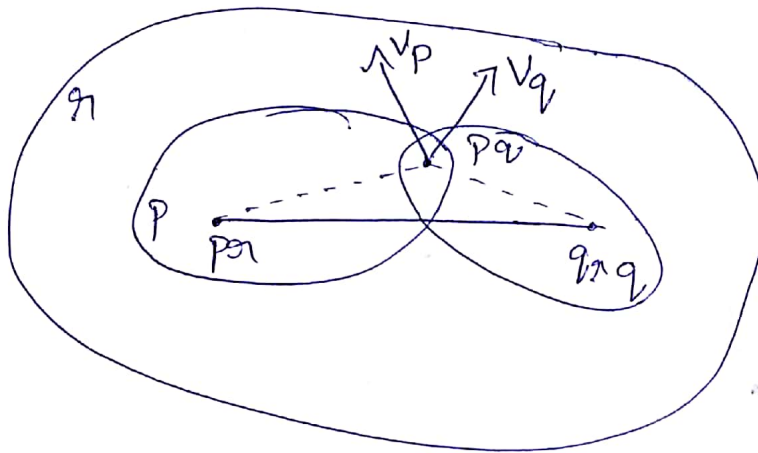
$$\therefore N = \frac{n(n-1)}{2}$$

Where  $N$  = Number of I-centres.

$n$  = number of bodies or links.

## Kennedy's Theorem:-

Consider three plane bodies  $p$ ,  $q$  and  $a$ ;  $a$  being a fixed body.  $p$  and  $q$  rotate about centre  $p_a$  &  $q_a$  respectively relative to the body  $a$ . Thus,  $p_a$  is the I-centre of bodies  $p$  &  $a$  whereas  $q_a$  is the I-centre of bodies  $q$  &  $a$ . Assume the I-centre of the bodies  $p$  &  $q$  at the point  $pq$ .



If the I-centre  $pq$  is considered on the body  $p$ , its velocity  $v_p$  is perpendicular to the line joining  $pq$  &  $p_a$ .

If the I-centre  $pq$  is considered on the body  $q$ , its velocity  $v_q$  is perpendicular to the line joining  $pq$  &  $q_a$ .

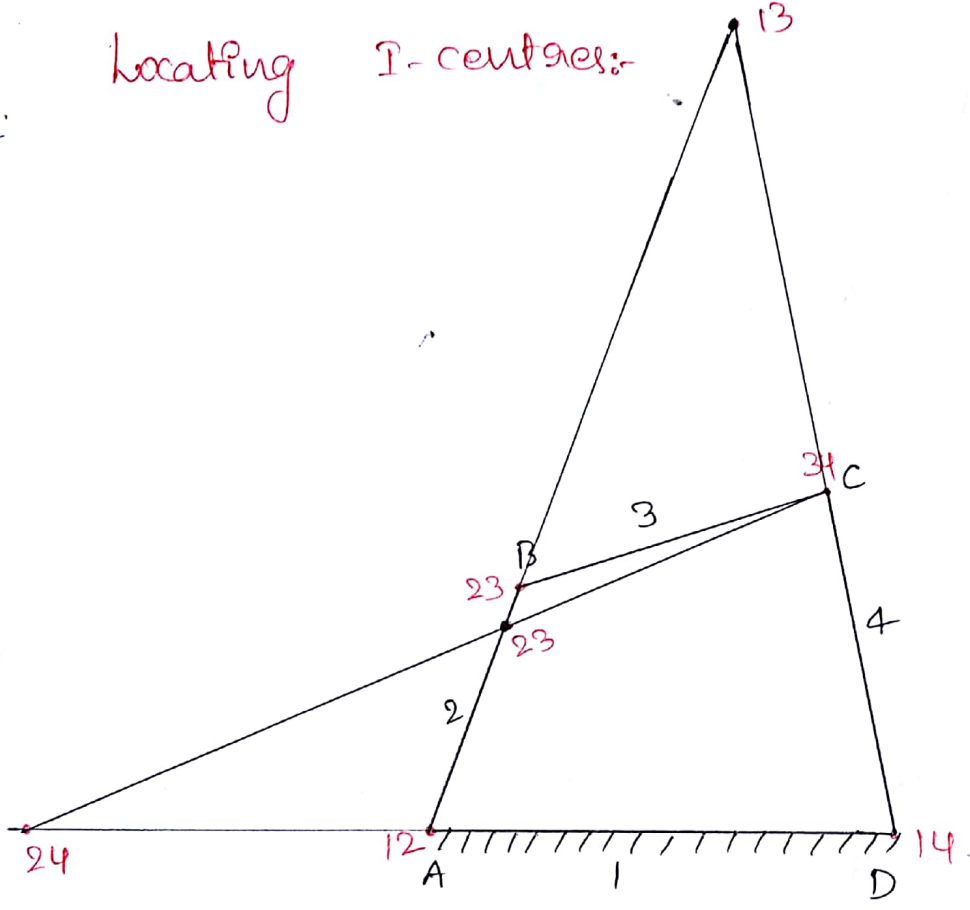
But it is impossible to have two velocities of the I-centre  $pq$  are in different directions. Therefore, the I-centre of the bodies  $p$  &  $q$  cannot be at assumed position  $pq$ . The velocities  $v_p$  &  $v_q$  of the I-centre will be same only



Qc If this centre lies on the line joining p<sub>31</sub> and q<sub>31</sub>.

11 Kennedy's theorem states that, if three plane bodies have relative motion among themselves, their I-centres must lie on a straight line.

Locating I-centres:-



$$N = \frac{n(n-1)}{2}$$

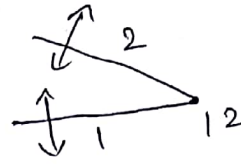
$$N = \frac{4(4-1)}{2}$$

$$N = 6$$

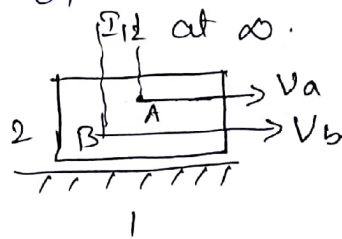
12 and 14 are fixed I-centres.  
 23 and 34 are permanent I-centres.  
 13 and 24 are neither fixed nor permanent but can be located easily by applying Kennedy's theorem.

## Rules to locate I-centres by inspection:-

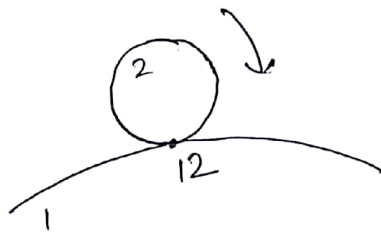
(1) In a pivoted joint, the centre of the pivot is the I-centre for the two links of the pivot.



(2) In a sliding motion, the I-centre lies at infinity in a direction perpendicular to the path of motion of the slider.



(3) In a pure rolling contact of the two links, the I-centre lies at the point of contact at the given instant.



## Angular velocity Ratio Theorem:-

When the angular velocity of a link is known and it is required to find the angular velocity of another link, locate their common I-centre.

For example, if it is required to find the angular velocity of link 4 when the angular velocity of the link 2 of a four-bar mechanism is known, locate I-centre 24.

Then

$$V_{24} = \omega_2 (24-12)$$

$$V_{24} = \omega_4 (24-14)$$

$$\Rightarrow \omega_4 = \frac{V_{24}}{24-14}$$

$$\omega_4 = \frac{\omega_2 (24-12)}{24-14}$$

$$\therefore \boxed{\frac{\omega_4}{\omega_2} = \frac{24-12}{24-14}}$$

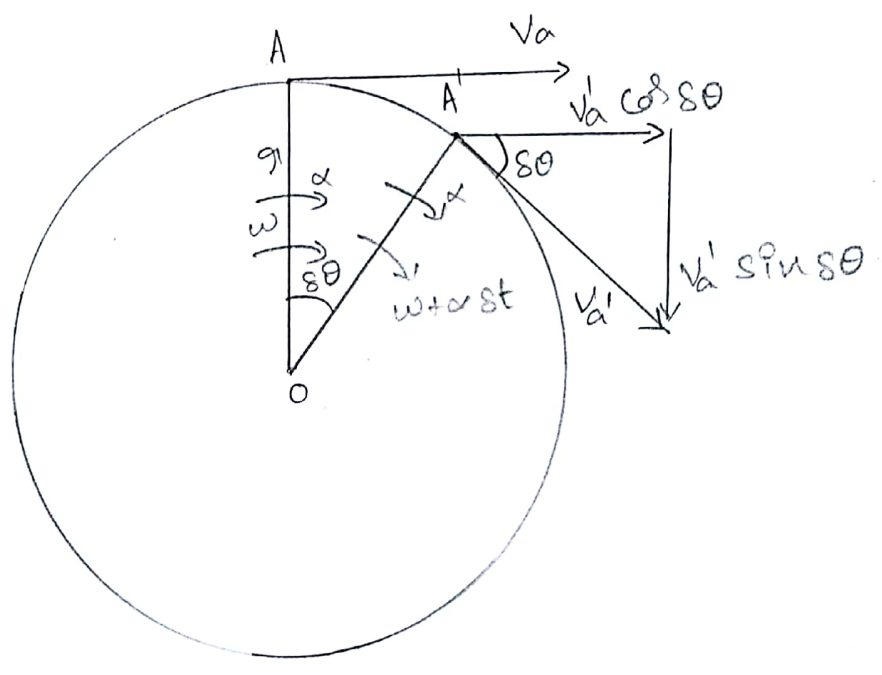
This is known as the angular-velocity-ratio-theorem.

$\therefore$  The angular velocity ratio of two links relative to a third link is inversely proportional to the distances of their common I-centre from their respective centres of rotation.

### Acceleration Analysis:-

Acceleration:- The rate of change of velocity w.r.t time is known as acceleration and it acts in the direction of the change in velocity. It is a vector.

Let a link OA, of length r, rotate in a circular path in the clockwise direction.



It has an angular velocity  $\omega$  and an angular acceleration  $\alpha$  in the same direction, i.e. the angular velocity increases in the clockwise direction.

Tangential velocity of A,  $V_a = \omega r$ .

In a short interval of time st, OA will be at OA' by rotating through a small angle sθ.

Angular velocity of OA',  $\omega_a' = \omega + \alpha st$ .

Tangential velocity of A',  $V_a' = (\omega + \alpha st)r$ .

Tangential velocity of A' will have two components.



change of velocity perpendicular to OA :-

velocity of A  $\perp$  to OA =  $v_a$ .

velocity of A'  $\perp$  to OA =  $v_a' \cos \theta$ .

$\therefore$  change of velocity =  $v_a' \cos \theta - v_a$ .

Acceleration of A  $\perp$  to OA =  $\frac{(\omega + \alpha st) r \cos \theta - \omega r}{st}$

As  $st \rightarrow 0$ ,  $\cos \theta \rightarrow 1$

$$\begin{aligned} \therefore \text{Acceleration of A } \perp \text{ to OA} &= \frac{\omega r - \omega r}{st} + \frac{\alpha st \cdot r}{st} \\ &= \alpha \cdot r \\ &= \left( \frac{d\omega}{dt} \right) r. \end{aligned}$$

$$= \frac{d}{dt}(\omega r)$$

Acceleration of A  $\perp$  to OA,  $f_{ao}^t = \frac{dv}{dt}$ .

$$\boxed{\therefore f_{ao}^t = \frac{dv}{dt}}$$

This is known as tangential acceleration of A relative to O.

change of velocity parallel to OA :-

velocity of A parallel to OA = 0.

velocity of A' parallel to OA =  $v_a' \sin \theta$ .

$\therefore$  change of velocity =  $v_a' \sin \theta - 0$   
 $= v_a' \sin \theta$ .

Acceleration of A parallel to OA =  $\frac{(\omega + \alpha st) r \sin \theta}{st}$

As  $st \rightarrow 0$ ,  $\sin \theta \rightarrow \theta$ .

Acceleration of A parallel to OA =  $\omega r \cdot \frac{d\theta}{dt}$

=  $\omega r \cdot \omega$

=  $\omega^2 r$

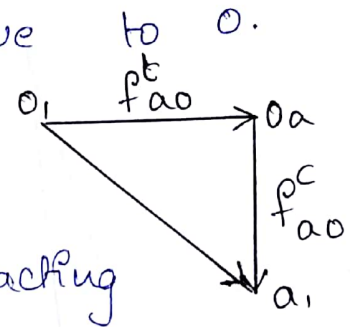
=  $\frac{v^2}{r^2} \cdot r$

$f_{ao}^c = \frac{v^2}{r}$



This acceleration is known as centripetal or radial acceleration of A relative to O.

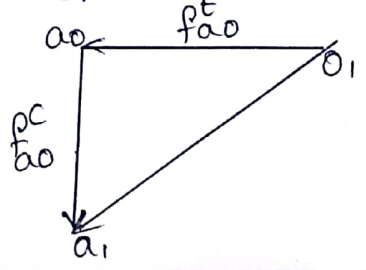
This figure shows the centripetal and the tangential components of the accelerations acting on A.



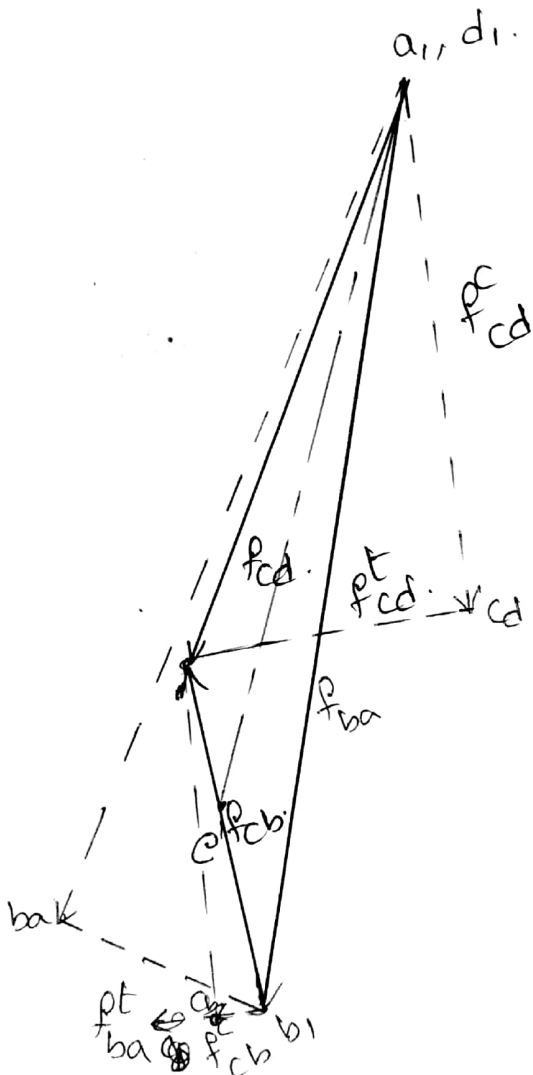
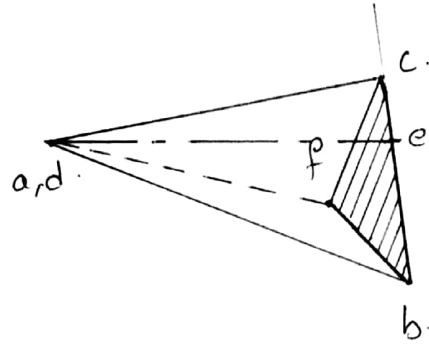
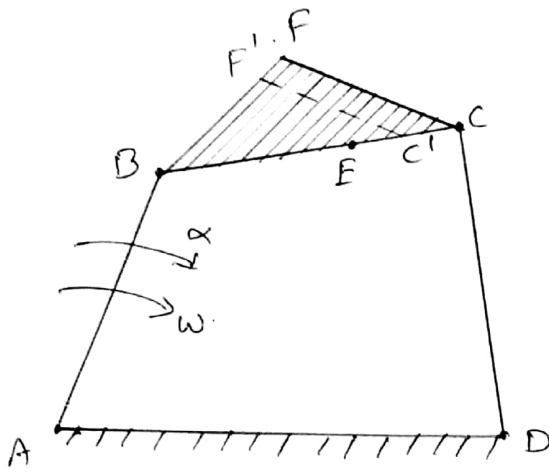
→ when  $\alpha = 0$ , i.e. OA rotates with uniform angular velocity,  $f_{ao}^t = 0$  and thus  $f_{ao}^c$  represents total acceleration

→ when  $\omega = 0$ , i.e. A has a linear motion,  $f_{ao}^c = 0$  and thus the tangential acceleration is total acceleration.

→ when  $\alpha$  is negative or the link OA decelerates, tangential acceleration will be negative. or it



Four-Link Mechanism :-



## Construction:-

(3)

- select the point  $a_1$  or  $d_1$ .
- Take the vector  $a_1 b_1$  to convenient scale in the proper direction and sense.
- Add the second vector to the first and then the third vector to the second.
- For the addition of the fourth vector, draw a line perpendicular to BC through the head  $c_3$  of the third vector. The magnitude of the fourth vector is unknown and  $c_1$  can lie on either side of  $a_1$ .
- Take the fifth vector from  $d_1$ .
- For the addition of the 6th vector to 5th, draw a line  $\perp DC$  through the head  $g$  of the 5th vector. The intersection of this line with the line drawn in 4th step locates the point  $c_1$ .

Total acceleration of B =  $a_1 b_1$

Total acceleration of C rel. to B =  $b_1 c_1$

Total acceleration of C =  $d_1 c_1$

## Angular Acceleration of Links:-

Tangential acc. of B rel. to A,  $\frac{pt}{t_{ba}} = \alpha \cdot AB = \alpha \cdot BA$

Tangential acc. of C rel. to B,  $\frac{pt}{t_{cb}} = c_b c_1$

$$\text{As } \frac{pt}{t_{cb}} = \alpha_{cb} \cdot CB$$

$$\Rightarrow \alpha_{cb} = \frac{\frac{pt}{t_{cb}}}{CB}$$



Let  $\alpha$  = angular acceleration of AB at this instant, assumed positive, i.e. the speed increases in the clockwise direction.

Acc of C rel to A = Acc of C rel to B + Acc of B rel to A

$$f_{ca}^c = f_{cb}^c + f_{ba}^c$$

$$\text{or } f_{cd}^c = f_{ba}^c + f_{cb}^c$$

$$d_1 c_1 = a_1 b_1 + b_1 c_1$$

$$f_{cd}^c + f_{cd}^t = f_{ba}^c + f_{ba}^t + f_{cb}^c + f_{cb}^t$$

$$d_1 c_1 + c_1 d_1 = a_1 b_1 + b_1 a_1 + b_1 c_1 + c_1 b_1$$

S.No	Vector	Magnitude	Direction	Sense
1.	$f_{ba}^c$ or $a_1 b_1$	$\frac{(ab)^v}{AB}$	$\parallel AB$	$\rightarrow A$
2.	$f_{ba}^t$ or $b_1 a_1$	$\alpha \times AB$	$\perp AB$ or $\parallel ab$	$\rightarrow b$
3.	$f_{cb}^c$ or $b_1 c_1$	$\frac{(bc)^v}{BC}$	$\parallel BC$	$\rightarrow B$
4.	$f_{cb}^t$ or $c_1 b_1$	-	$\perp BC$	-
5.	$f_{cd}^c$ or $d_1 c_1$	$\frac{(dc)^v}{DC}$	$\parallel DC$	$\rightarrow D$
6.	$f_{cd}^t$ or $c_1 d_1$	-	$\perp DC$	-

Tangential acc. of c rel. to D,  $f_{cd}^t = \alpha_{cd} \cdot CD$ . (4)

$$f_{cd}^t = \alpha_{cd} \cdot CD$$

$$\Rightarrow \alpha_{cd} = \frac{f_{cd}^t}{CD} = \frac{CD \cdot \alpha_c}{CD}$$

Acceleration of intermediate and offset points:-

Intermediate Point:- The acceleration of intermediate points on the links can be obtained by dividing the acceleration vectors in the same ratio as the points divide the links. For point E on BC,

$$\frac{BE}{BC} = \frac{b_1 e_1}{b_1 c_1}$$

$a_{e_1}$  gives the total acceleration of point E.

Offset points:-

$$f_{fa}^p = f_{fb}^p + f_{ba}$$

$$= f_{ba} + f_{fb}^p$$

$$= f_{ba} + f_{fb}^{pc} + f_{fb}^{pt}$$

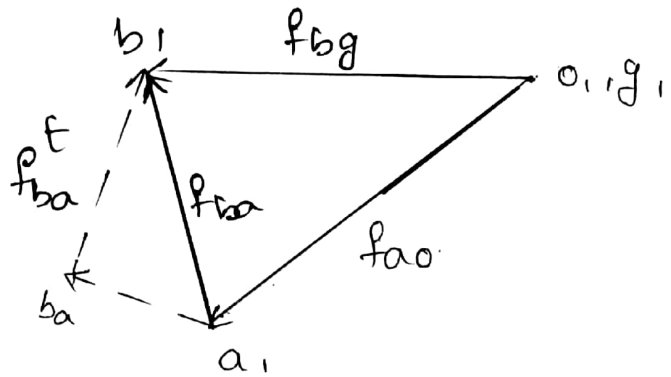
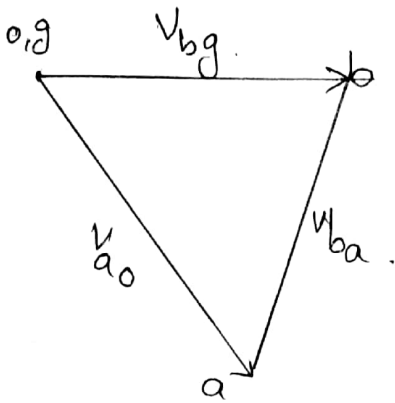
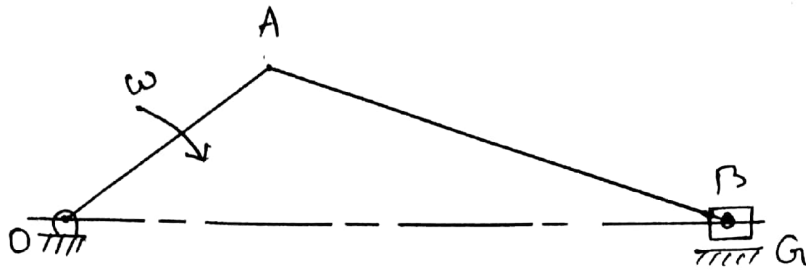
$$a_{f_1} = a_{b_1} + b_1 f_b + f_b f_1$$

$$f_{fb}^{pc} = \frac{(bf)^v}{BF}, \parallel FB$$

$$f_{fb}^{pt} = \alpha_{fb} \times FB = \alpha_{cb} \times FB$$

$$= \frac{f_{cb}^{pt}}{CB} \times FB, \perp \text{ to } FB$$

# Slider-Crank Mechanism:-



Acc. of B rel. to O = Acc. of B rel. to A + Acc. of A rel. to O.

$$f_{bo} = f_{ba} + f_{ao}$$

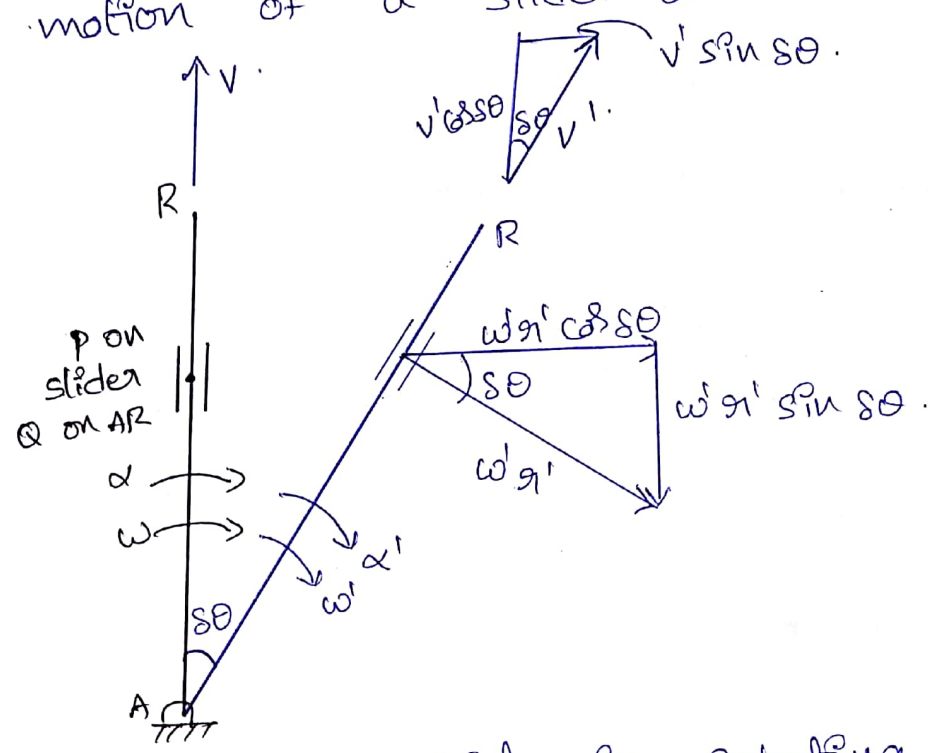
$$f_{bg} = f_{ao} + f_{ba}^c + f_{ba}^t$$

$$g, b_1 = o, a_1 + a_1 b_a + b_a b_1$$

S.No	vector	Magnitude	Direction	Sense
1.	$f_{ao} \propto o, a_1$	$\frac{(oa)^2}{OA}$	$\parallel OA$	$\rightarrow O$
2.	$f_{ba}^c \propto a_1, b_1$	$\frac{(ab)^2}{AB}$	$\parallel AB$	$\rightarrow A$
3.	$f_{ba}^t \propto b_a b_1$	-	$\perp AB$	-
1.	$f_{bg} \propto g_1, b_1$	-	$\parallel$ to B	-

Coriolis Acceleration:-

In some cases, the point may have its motion relative to a moving body system, for example, motion of a slider on a rotating link.



Consider a link AR, which is rotating about a fixed point A. P is a point on a slider Q on the link AR.

At any given instant,

- $\omega$  = angular velocity of the link.
- $\alpha$  = angular acceleration of the link.
- $v$  = linear velocity of the slider on the link
- $f$  = linear acceleration of the slider on the link.
- $r$  = radial distance of point P on the slider

After a short interval of time  $st$ , let

- $\omega' = \omega + \alpha st$  = angular velocity of link.
- $v' = v + f st$  = linear velocity of the slider on link.
- $r' = r + s r$  = radial distance of the slider.



Acceleration of P Parallel to AR :-

Initial velocity of P along AR =  $v = v_{pq}$ .

Final velocity of P along AR =  $v' \cos \theta - \omega' r' \sin \theta$

change of velocity along AR =  $v' \cos \theta - \omega' r' \sin \theta - v$

Acceleration of P along AR

$$= \frac{(v + f \Delta t) \cos \theta - (\omega + \alpha \Delta t) (r + \Delta r) \sin \theta - v}{\Delta t}$$

As  $\Delta t \rightarrow 0$ ,  $\cos \theta \rightarrow 1$ ,  $\sin \theta \rightarrow \theta$ .

Acceleration of P along AR =  $f - \omega r \frac{d\theta}{dt}$

$$= f - \omega^2 r$$

= Acc. of slider - centripetal acc

Acceleration of P perpendicular to AR :-

Initial velocity of P  $\perp$  to AR =  $\omega r$ .

Final velocity of P  $\perp$  to AR =  $v' \sin \theta + \omega' r' \cos \theta$

change of velocity  $\perp$  to AR =  $v' \sin \theta + \omega' r' \cos \theta - \omega r$

Acceleration of P  $\perp$  to AR

$$= \frac{(v + f \Delta t) \sin \theta + (\omega + \alpha \Delta t) (r + \Delta r) \cos \theta - \omega r}{\Delta t}$$

As  $\Delta t \rightarrow 0$ ,  $\cos \theta \rightarrow 1$ ,  $\sin \theta \rightarrow \theta$ .

Acceleration of P  $\perp$  to AR =  $v \frac{d\theta}{dt} + \omega \frac{dr}{dt} + r \alpha$

$$= v\omega + \omega v + r\alpha$$

$$= 2v\omega + r\alpha$$

Acceleration of  $P \perp$  to  $AR = 2\omega v + \text{tangential acc.}$  ⑥

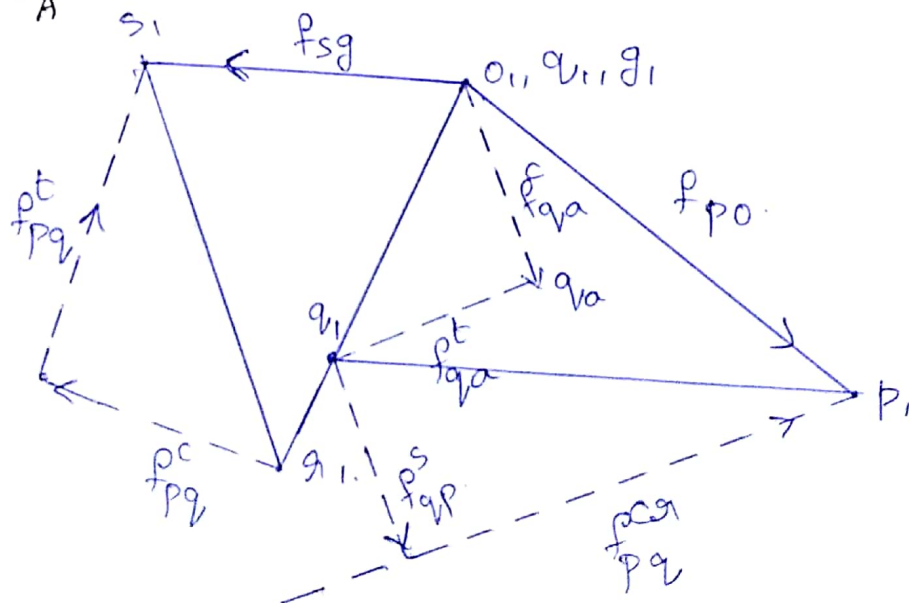
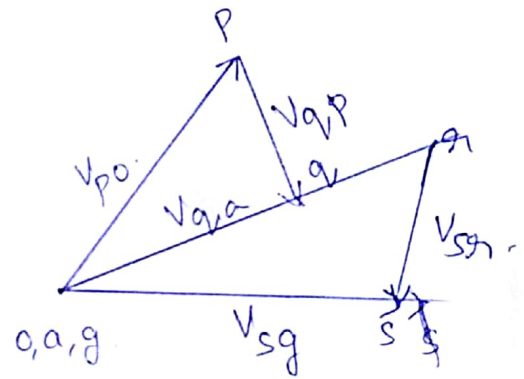
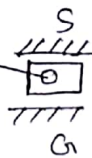
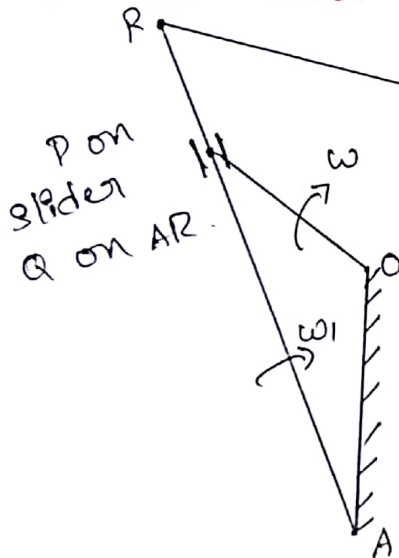
The component  $2\omega v$  is known as Coriolis acceleration.

Coriolis component is positive if

→ Link  $AR$  rotates clockwise and the slider moves radially outwards.

→ Link rotates counter-clockwise and the slider moves radially inwards.

### Crank and slotted lever mechanism:-



The crank OP rotates at uniform angular velocity of  $\omega$  rad/s clockwise.

$$\text{or } f_{pa} = f_{pq} + f_{qa}$$

$$\text{or } f_{qo} = f_{qp} + f_{po}$$

$$\text{or } f_{po} = f_{qa} + f_{pq}$$

$$f_{po} = f_{qa}^c + f_{qa}^t + f_{pq}^s + f_{pq}^{ca}$$

$$o, p, i = a, q, a + q, a, q, i + q, i, p, q + p, q, p, i$$

S.No	Vector	Magnitude	Direction	Sense
1.	$f_{po}$ or $o, p, i$	$\omega \times OP$	$\parallel OP$	$\rightarrow O$
2.	$f_{qa}^c$ or $a, q, a$	$\frac{(aq)^{\omega}}$ AQ	$\parallel AQ$	$\rightarrow A$
3.	$f_{qa}^t$ or $q, a, q, i$	-	$\perp AQ$	-
4.	$f_{pq}^s$ or $q, i, p, q$	-	$\parallel AR$	-
5.	$f_{pq}^{ca}$ or $p, q, p, i$	$2\omega, v_{pq}$ $= 2\left(\frac{aq}{AQ}\right)v_P$	$\perp AR$	-

$\omega_1 =$  angular velocity of AR  $\odot \odot$

$$\omega_1 = \frac{aq}{AQ}$$